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A MODEL OF V356 SAGITTARII

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ABSTRACT

A model is advanced to explain the peculiar features of the unusual eclipsing binary V356 Sgr, for which previous analysis has not been very successful. The most obvious peculiarities were pointed out by Popper and consist of the absence of a significant reflection effect and anomalous depths for the eclipses. However, additional strange features are also present. The light curves cannot be satisfied with a conventional model, even if one allows for the fast rotation of the primary. In our model, a thick, opaque ring or disk of recently transferred matter surrounds the present primary star. The ring is rather substantial and is *not* similar to the emission-line rings in systems such as RW Tau and U Cep. We have produced a first-order, but quantitative, computer model of the disk (along with the rest of the system) and have obtained its approximate parameters by the method of differential corrections. We account for the presence of the disk as a consequence of the known fast rotation of its central star, and we propose that the same condition accounts for the disk in the β Lyr system, where the central star is not directly visible. Numerous observational arguments indicate that V356 Sgr corresponds to a later evolutionary stage of a system like β Lyr, as was earlier suggested by Woolf. The system fills a gap in the observational sequence of examples of binary star evolution, and must be between the β Lyrae stage and a later detached stage, with no disk, no mass transfer, and a helium-star secondary. Probably mass transfer has been terminated by helium ignition in the present secondary, but there is a small chance that a slow mass transfer still goes on. Part of the (possible) underluminosity of the primary indicated earlier is removed by our model. The rest (about 1 mag) could be accounted for, if the primary is of spectral type B1 rather than B3, which may be within the range of uncertainty of classification.

Subject headings: stars: eclipsing binaries — stars: individual

I. BACKGROUND

V356 Sgr is an abnormal member of the Algol class of binaries. According to Popper (1955), the primary component is of spectral type B3 V and is rotating rapidly, while the secondary is of type A2 II and is rotating at least approximately in synchronism with the orbital motion. Both old and new analyses of the light curves indicate that the secondary is about the size of its Roche lobe, so that the system is either semidetached or quite near to being semidetached. The binary has been cited by several authors (e.g., Popper 1955; Woolf 1965; Stothers and Lucy 1972; Stothers 1973) for its evolutionary relation to other unusual binaries such as β Lyr, μ^1 Sco, and SX Aur. Stothers and Lucy (1972) proposed the primary component as a possible example of a star which is strongly underluminous because it is in rapid differential rotation, with the core spinning much faster than the envelope. This would be interesting, if true, because observational counterparts of the differentially rotating model stars computed by Bodenheimer and Ostriker (1970) and by Bodenheimer (1971) are currently lacking, or at least are rare (viz., Hall 1971; Popper and Plavec 1976). Also, Stothers (1973) included V356 Sgr in a rather thorough discussion of the role of rotation in close binary systems.

Thus for several reasons it would be good to have an accurate description of the system of V356 Sgr, and that is the object of this paper. However, the light curves (Popper 1957) have anomalies which show the presence of some major peculiarity. The most important of these anomalies were recognized and discussed by Popper (1955) in an admirable attempt to understand the system within the limitations of the solution techniques of 20 years ago. The main anomalies may be listed as follows:

1. The ratio of eclipse depths, which to first order depends only on the components' temperatures, is less than would be indicated by the spectral types. Roughly speaking, primary eclipse is a little more than twice as deep as secondary, but the ratio should be considerably larger. This can be appreciated from Figure 1, where a light curve based essentially on Popper's parameters for V356 Sgr is compared with the V light curve. The situation is made worse by the

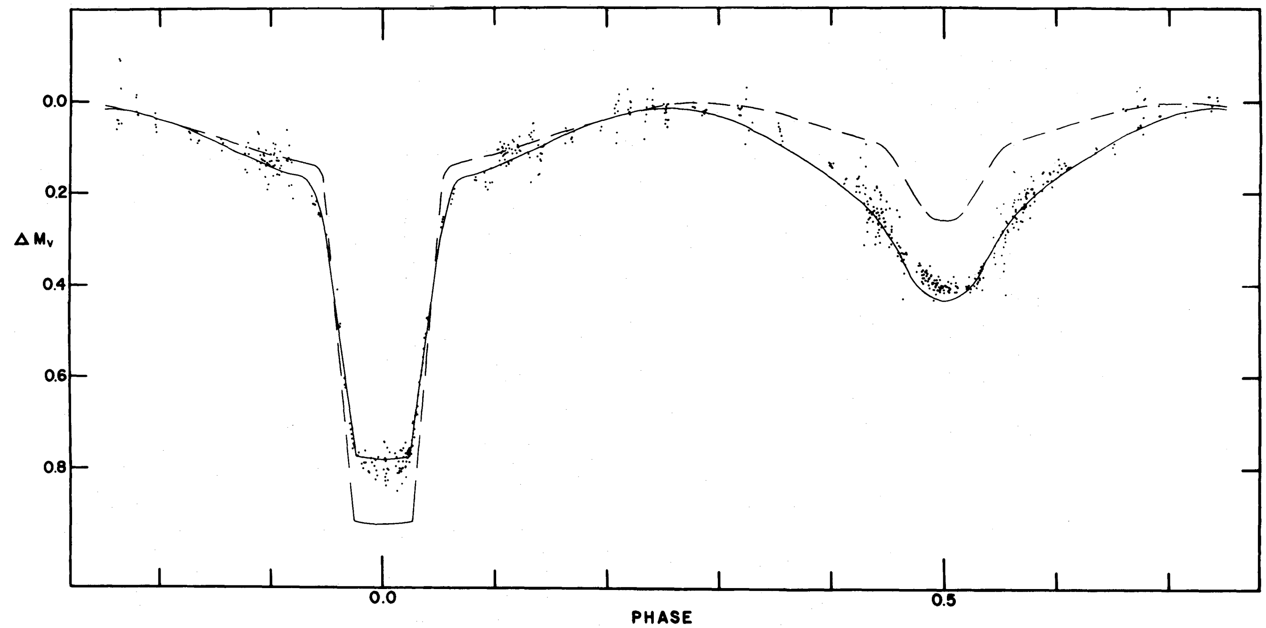

FIG. 1.—Popper's *V* observations of V356 Sgr. The dashed curve is based essentially on Popper's parameters (no disk, of course). The continuous curve was computed from the present model (with disk).

gravity darkening which must be present on the tidally distorted secondary. Thus, in the (annular) secondary eclipse, it is mainly the relatively dim region of the A2 star near the inner Lagrangian point which is eclipsed, so that not only the relatively low temperature of the secondary but also this latter circumstance predict a shallow secondary eclipse. Nevertheless, secondary eclipse is fairly deep. There would seem to be no chance that the spectral types could be enough in error to account for this difficulty.

2. The light curves show only a very small reflection effect. Not only the difference in heights of the eclipse shoulders but also the strong curvature in the maxima show the absence of a reflection effect. That is, the cos 2θ Fourier component of reflection has an opposite sign to that for ellipticity and tends to cancel the apparent ellipticity effect. Since the eclipse durations establish the approximate dimensions of the stars, one can calculate (as did Popper) the expected amount of reflection. Popper found that his estimated *upper limit* on the observed (differential) reflection was less than half that expected theoretically. We find (Table 1) that the observed amount is only of the order of 1/10 of the theoretical amount. How can the reflection effect simply fail to appear in the light curve?

3. The duration of secondary eclipse appears to be greater than that of primary eclipse, although any orbital eccentricity must be quite small; thus the durations should be sensibly equal.

4. The detailed character of primary eclipse is strange, in that the bottom, which according to preliminary analysis must be total, does not appear quite flat, while the shoulders of the eclipse show peculiar "ledges" before and after the eclipse proper. The curvature in the bottom of eclipse is greater than could be accounted for by ellipsoidal variation.

The most obvious and undeniable of these are points 1 and 2 (those noted by Popper), but points 3 and 4 are also interesting. In trying for a consistent solution,

TABLE 1

DESCRIPTION OF V356 SAGITTARII

A. FIXED PARAMETERS

g_1	1.00	$\mathfrak{M}_2/\mathfrak{M}_1$	0.3884
g_2	1.00	$x1_V$	0.37
T_1	17900 K	$x2_V$	0.55
A_1	0.90	$x1_B$	0.51
Ω_2	2.654	$x2_B$	0.70
a	0.08		

B. ADJUSTED PARAMETERS

i	86°6	$L_1/(L_1+L_2)_V$	0.483
T_2	10340 K	$L_1/(L_1+L_2)_B$	0.548
Ω_1	7.549	c	0.04
r_d	0.23	A_2	0.08

C. RELATIVE RADII

Synchronous (from adjustment):		Primary rotating at limiting rate (not from actual solution):	
r_1*(pole)	0.140	r_1(pole)	0.111
r_2(pole)	0.280	r_1(point)	0.161
r_2(point)	0.404	r_1(side)	0.158
r_2(side)	0.292	r_1(back)	0.160
r_2(back)	0.325		

* Only the polar value is listed, since the star is nearly spherical in synchronous rotation.

Popper considered a number of possibilities which cannot be discussed for lack of space, but his main hypothesis was that the A2 star has an extended atmosphere and that its limb darkening is nonlinear and very large (having a coefficient greater than unity, if approximated by the linear law). Obviously, a large limb darkening will counteract the effect of gravity darkening on the depth of secondary eclipse. He then concluded that satisfactory agreement with the light curves could be attained and that the surface brightness ratio of the components could be made to agree with their spectral types. Figure 1 shows that Popper's solution parameters, coupled with Planckian surface brightnesses appropriate to the spectroscopic temperatures, do not reproduce the observed light curves (we set the limb-darkening coefficient equal to unity and did not include extended atmosphere effects). Although Popper advocated an extended atmosphere and an effective limb-darkening coefficient *greater than* unity, it seems unlikely that those features could bring about agreement with the observations. Popper did not publish a computed light curve. Thus the original problem contained in anomalies 1 and 2 remains. Incidentally, in Popper's solution, the inclination was fixed at 90°, because preliminary trials gave $\cos^2 i < 0$.

We can be sure that a fundamental physical peculiarity is associated with V356 Sgr, because the eclipses are obviously total-annular and both are reasonably deep; so the solution would normally be very well determined. The lack of agreement between theory and observation can be due only to an important attribute of the binary which has not yet been incorporated into the theory.

II. OBSERVATIONAL BASIS FOR UNDERSTANDING V356 SAGITTARII

The V and B light curves of V356 Sgr published by Popper (1957) are shown in Figures 1 and 2, respectively. The phase coverage is good despite the long period of 8.9 days. Masses of 12.1 and 4.7 $\mathfrak{M}_\odot$ were found by Popper (1955) for the primary and secondary components. The double-lined velocity curve appears to be fairly good but could be improved upon, according to Popper. The main difficulty lies in the rotational broadening of the primary's lines. Popper stated that the primary rotates much faster than synchronously, and Olson (1977) gives $V \sin i \approx 260$ km s^{-1}.[1] Because of the interesting nature of this system as a check on binary star evolution theory, it seems quite important that the following observational problems be undertaken:

a) The mass ratio especially and, to a lesser extent, the absolute masses require determination to high accuracy.

b) The system should be monitored occasionally to search for emission-line activity associated with mass transfer. To our knowledge, no one has reported spectroscopic evidence for mass transfer to date. According to Popper (1977), no emission lines are visible on his spectra, although Hα was not observed. Plavec and Polidan (1976) report no emission-line activity at Hα.

c) Times of minima are needed to check for period changes associated with mass transfer. The system is a

[1] See also discussion on the rotational velocity in § V.

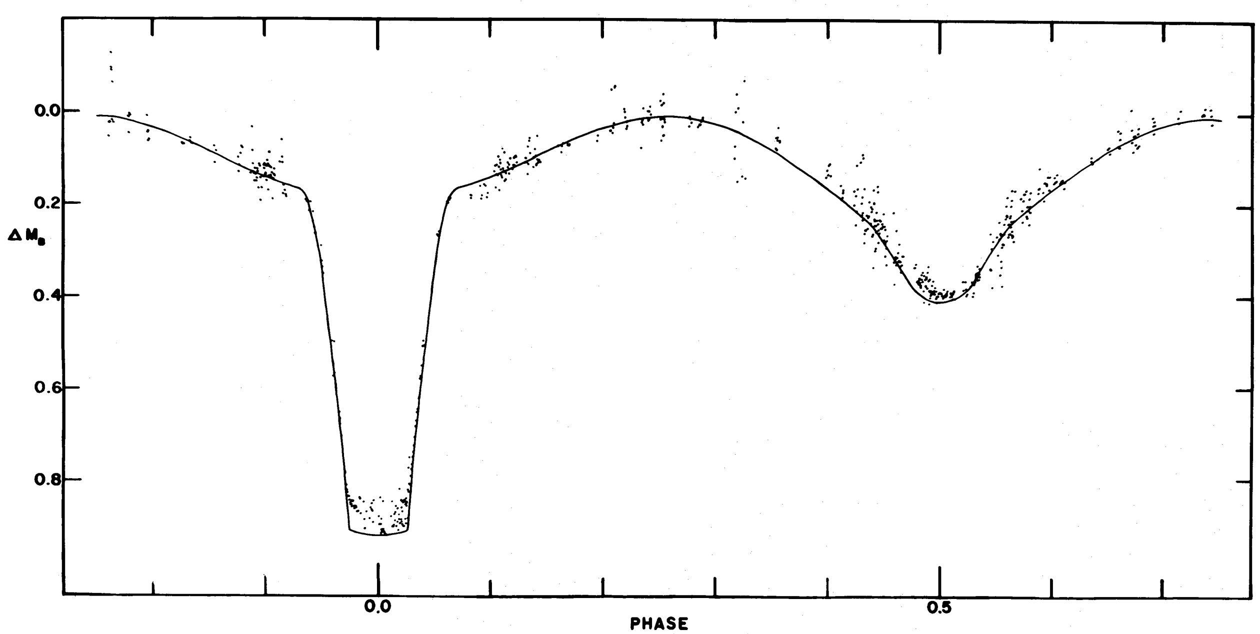

FIG. 2.—Popper's B observations and a computed 4400 Å light curve for our model

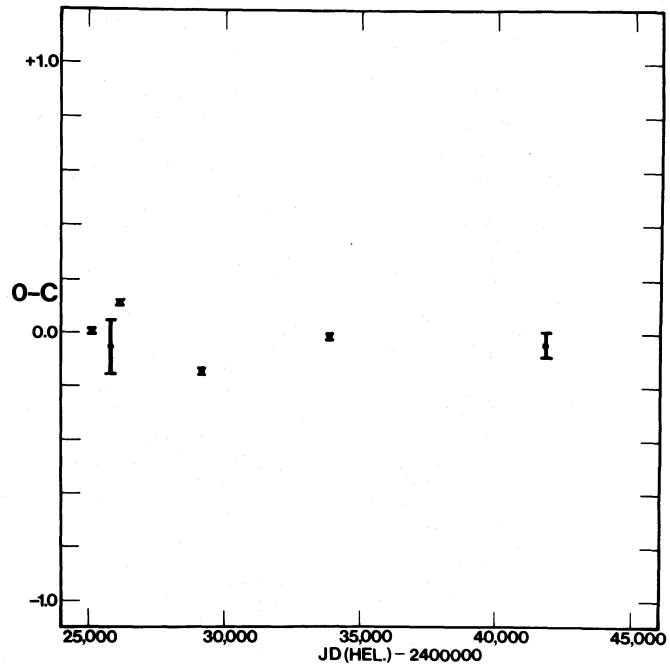

FIG. 3.—Residuals from the ephemeris: $T_{\min}$ = HJD 2,425,111.827 + 8.89610E days. The error bars on the last point are from our estimate of the maximum error. The other bars are based simply on the number of digits published and may be quite unrealistic.

difficult one for times of minima, because the eclipse durations are more than a day. Figure 3 shows the few points available for an $O - C$ diagram. Four of these were kindly communicated by J. M. Kreiner (Cracow), who collected them from four publications and one private communication. Another was published by Dworak (1977). The sixth point was found by us from recent photoelectric observations made by E. J. Woodward (private communication). We used a graphical sliding fit method to find the phase shift of the entire light curve with respect to Popper's (1955) ephemeris. The error bars in Figure 3 are our estimates (since, except for Dworak, the original authors provided no error estimates) and are based simply on the number of digits given. In our opinion there is no significant evidence for a period change in the data now available—and certainly not a significant secular change.

Because of a lack of information of types (*b*) and (*c*) above, we do not know for certain that all mass transfer has stopped and that the secondary component is now detached from its Roche lobe. Conceivably, the system could be in a brief period of slow mass transfer (following the end of the rapid phase). While only a trifling amount of mass would be exchanged in such a short interval (cf. Plavec 1968 for time scales in case B mass exchange) of slow transfer, the process could pinpoint very accurately the stage of evolution of the binary. The interval should end with the ignition of helium in the core of the secondary, and the observational question posed here concerns whether this has already happened.

III. THE DISK

Since Popper's well thought-out attempts at a solution were, in the end, unsuccessful (except to discover and underscore the difficulties of the problem), we believe that the correct solution must incorporate some major, and previously unaccounted for, feature. Specifically, we propose that a thick, opaque circumstellar disk of recently transferred material surrounds the primary component. Its effective temperature is low enough (~6000–7000 K) that we may, for the present calculations, neglect its surface brightness compared with that of the B3 star. This disk blocks a substantial fraction of the primary's light and causes it to appear underluminous for its mass. However, the disk does not alter the star's spectral type. This idea would have been radical 20 years ago but is not at all so today, in view of the somewhat similar disk around β Lyr (Huang 1963; Wilson 1974). The effects of such a disk may be visualized with the help of Figure 4. Since the disk reduces the light of the primary, it should make primary eclipse shallower than expected (observational anomaly 1). For the same reason it will greatly reduce the irradiation of the secondary by the primary, and thus will reduce the reflection effect (anomaly 2). If the disk emits relatively little light of its own, the effect of its eclipse by the secondary star will be slight, but the eclipse of the secondary by the disk should be appreciable. Thus secondary eclipse should be wider than primary eclipse (anomaly 3), and it should also be deeper than it would be without an eclipse by the disk (anomaly 1). However, the slight

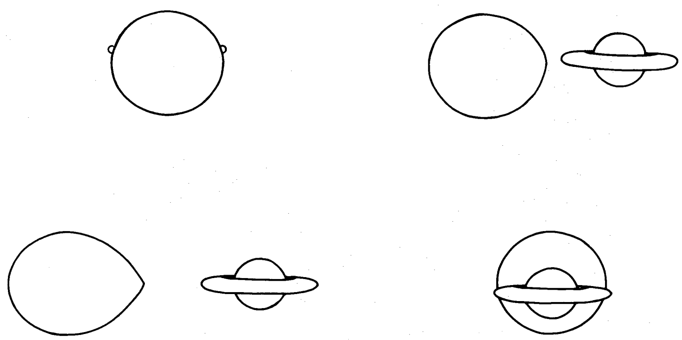

FIG. 4.—Scale drawing of V356 Sgr, according to the results of Table 1. The phases shown are 0.000. 0.125, 0.250, and 0.500.

drop in light caused by the eclipse of the disk may be marginally detectable (anomaly 4).

We did attempt to fit the light curves without invoking a disk and were not able to achieve even rough agreement when the temperatures were required to have the spectroscopic values. These trials used the general procedure for computing light curves of tidally distorted eclipsing binaries developed by Wilson and Devinney (1971). Unless one is willing to consider the possibility that at least one of the published spectral types is completely wrong (i.e., not even close—and there is no reason to suspect this), some means for reducing the light of the primary *without* reducing its surface brightness is required. The most obvious way to do this is with a thick, opaque disk around the primary, lying in the orbit plane of the binary. This would accomplish the desired effect by geometrical blocking of part of the primary's light and would also deepen secondary eclipse considerably. The disk must be thick, because a disk of negligible thickness would reduce the brightness of the primary only slightly for orbital inclinations close to 90°.

We therefore decided that it is essential to include the effects of a thick disk in computing light curves of V356 Sgr. However, it is not at once clear whether the disk is in full contact with the primary star or whether it is in the form of a detached or partly detached ring (i.e., a torus, or "donut"). We first developed a computer light curve program which treats the two stellar components in the same way as does the regular Wilson-Devinney program but includes, in addition, a "disk" around the primary which has the same basic properties as that modeled for β Lyr by Wilson (1974). We did not include the effects of faster-than-synchronous rotation on the polar flattening of the primary star. In brief, the disk has the form of a flattened ellipsoid of revolution and is self-luminous, with a distribution of surface flux which, to first order, obeys a normal (von Zeipel) gravity-darkening law. The polar dimension of the disk is considerably less and the equatorial dimension is considerably more than that of the primary star, so that its surface lies partly inside and partly outside the star. The part which lies inside is ignored by the program, while the part which lies outside constitutes our thick disk. The disk conserves energy, in that all flux which enters through the star-disk interface is radiated at the surface of the disk. Thus the situation is qualitatively the same as for the Wilson (1974) model of β Lyr, except that here the higher-latitude regions of the primary star protrude through the disk.

We found that a self-luminous, energy-conserving disk of the type just described cannot account for the observed light curves of V356 Sgr. All computed light curves have very gradually rounded shoulders on the primary eclipse, whereas the observed primary eclipse begins and ends very abruptly (see Figs. 1 and 2). The problem is due to the fact that the surface brightness of the disk cannot differ greatly from that of the star near the place where the two surfaces interesect, since the disk must reemit all the energy supplied to it from below. Thus the eclipse of the disk itself *cannot* have only negligible photometric consequences, and this expectation is verified by the rounded ingress and egress of primary eclipse found in our computations.

If the disk is not in full contact with the primary star, its surface brightness need not be of the same order of magnitude as that of the star. In fact, the surface brightness for the outer facing surface of the disk can only be a free parameter at our present stage of understanding of the system, since we cannot compute it by any conservation principle and do not even know the approximate mass of the disk. In the interest of simplicity, therefore, we next adopted the extreme case of a completely nonluminous disk—one which eclipses but has no light of its own to be eclipsed. Certainly, the true case must lie between the two types of disk we have tried; but we leave that refinement for the future. The specific form adopted for our detached ring is that of an "elliptical torus." By this we mean a torus having elliptical meridian sections with semimajor axis a and semiminor axis c. Equilibrium configurations for self-gravitating toroidal figures (without central stars) have been studied by Wong (1974), who finds that stable equilibrium is expected over a wide range of relative dimensions, including those we find for V356 Sgr (Table 1) Of course, our case is different from that considered by Wong, in that there is a massive star at the center of the torus; but at least one cannot dismiss such figures as being obviously unstable.

IV. A QUANTITATIVE VIEW OF THE DISK

The light-curve solution was carried out by the method of differential corrections. The computer programs used were specially modified versions of the Wilson and Devinney (1971) light-curve and differential corrections programs. The modifications were similar to those used for analyzing β Lyrae light curves by Wilson (1974), except that the disk in this case is toroidal and its eclipse effects were computed by the procedure given by Wilson (1975, 1976). The equatorial plane of the disk (torus) coincides with the orbit plane of the binary. With these disk-eclipse calculations included, the programs run slowly, and it was necessary to use a rather small number of normal points (17 in each color). It did not seem worthwhile to develop a fast-running version of this program, since it was to be used on V356 Sgr only. Inclusion of the toroidal disk introduces three new adjustable parameters. These are a and c, the semimajor and semiminor axes of the elliptical meridian section of the torus, and r_d, the radius from the center of the primary star to the center of the meridian section (see Wilson 1975 for diagram). The equatorial radius for the disk is thus $a + r_d$, in units of the separation of the components. Since the disk is seen nearly edge-on, it is obvious that a and r_d cannot be determined separately in practice; so we fixed a and considered only r_d and c as adjustable disk parameters. With two new adjustable parameters added to the usual set, we run the risk of having too many parameters to treat realistically. Fortunately, there is a fairly good spectroscopic mass ratio q for the system, so we elected not to adjust q. We also *assumed* reasonable values (Table 1) for a number of other quantities. The value of 0.90 for the bolometric albedo of the primary star is an estimated *effective* albedo and is less than unity, because some shadowing of the primary by its own disk is expected. The Roche modified potential for component 2 (Ω_2) corresponds to exact filling of the secondary Roche lobe (semidetached condition).

The solution is given in Table 1. Although the program computes formal probable errors, they are not listed because they are unrealistically small and would give a false impression of the accuracy of the results. The parameters of Table 1 contain two important kinds of errors which are not accounted for in formal probable errors. These are systematic errors due to the first-order nature of the disk model and accidental errors due to the relative inaccuracy of numerical derivatives in this particularly tricky program. However, the numerical derivative scheme was checked very thoroughly for absence of simple programming errors, by computing the derivatives in two different ways.

Since we know that the primary component rotates faster than synchronously (Popper 1955; Olson 1977), we considered the possibility that the peculiarities of the system might be caused by rotational flattening and attendant rotational gravity darkening. That is, we asked whether the light curves could then be explained without resorting to a disk model. The light-curve program was modified to include the effects of nonsynchronous rotation, according to the equivalent theories by Plavec (1958), Limber (1963), or Kruszewski (1966). This was done *after* the solutions of Table 1 were completed. It seems that the most obvious of the several peculiarities, the absence of a reflection effect, *cannot* be explained by fast rotation. Consider, as a function of inclination, the ratio of the flux from the primary, rotating at its limiting[2] velocity, to the same quantity for a synchronously rotating (i.e., virtually spherical) star with the same 4π sr luminosity. Viewed from within its equatorial plane, we compute that the flux is still about 70% that from a spherical star. Therefore, rotational flattening could account for a reduction of the measured reflection effect to 70% of the normal amplitude, but it could not possibly account for the virtual absence of any reflection, as observed. We therefore did not pursue this idea any further. It seems that the disk is needed to prevent the radiation of the primary from reaching the "reflection effect cap" of the secondary. In principle, it would have been better to repeat the solutions with a disk *and* nonsynchronous rotation, but we felt this was not prudent with regard to the effective utilization of computer time. The results for disk and fast rotation should not differ much from those of Table 1, because the gravity-darkened equatorial zone of the primary would be hidden by the disk.

Theoretical light curves corresponding to the solution of Table 1 are graphed among Popper's observations in Figures 1 and 2. When one considers the oddities of the system, the agreement seems fairly good, although the depths of the eclipses are not in perfect register. However, it seems difficult to avoid the conclusion that a geometrically and optically thick disk, of the approximate dimensions given by Table 1, is present around the primary star. This disk accounts for the absence of a reflection effect on the secondary by blocking the primary's light. It also accounts for part of the apparent underluminosity of the primary by similarly blocking light from the observer. The disk also accounts for the fact that the secondary eclipse has a significantly greater duration than does the primary eclipse, because the disk has little self-luminosity. Thus eclipses *by* the disk are readily observable, but eclipses *of* the disk are not. However, the slight "ledges" on the shoulders of primary eclipse are probably caused by marginally detectable effects in the eclipse of the disk by the secondary star.

V. EVOLUTIONARY STAGE–OBSERVATIONAL CONSTRAINTS

The disk described in the previous section seems to be of the same general type as the disk in the β Lyr system. That is, it is geometrically thick and quite opaque, in contrast to some relatively insubstantial

[2] See § V for remarks on the limiting rotation rate.

disks, such as those in RW Tau and U Cep. We would like to know where the V356 Sgr disk stands in relation to the β Lyr disk, insofar as the evolution of the two binaries is concerned. Certainly, in each system the disk is a result of mass transfer due to Roche-lobe overflow, and there seems to be no reason to doubt that both systems are in the first epoch of mass exchange. We list below the main differences between the two disks, so that they may be examined for clues to the evolutionary state of V356 Sgr.

a) The V356 Sgr disk lies well within the primary Roche lobe, with $r_{\mathrm{disk}}/r_{\mathrm{lobe}} \approx 0.65$. The β Lyr disk essentially fills its lobe (Wilson 1974).

b) The thickness of the V356 Sgr disk is small enough to allow a good fraction of the primary star to be in view ($\sim 60\%$–65%), whereas the more massive component of β Lyr is completely hidden in its disk.

c) No spectroscopic evidence for mass transfer (emission lines with appropriate velocity behavior) has been reported for V356 Sgr, while such evidence is abundant for β Lyr (e.g., Sahade *et al.* 1959). Emission lines due to mass transfer should be easier to detect in V356 Sgr than in β Lyr because of the clear separation of the components in the former.

d) The mass ratio is less extreme in V356 Sgr (about 0.4) than in β Lyr (about 0.15).

Three of the four points above suggest clearly that V356 Sgr represents a later stage of the mass-exchange process than does β Lyr, as first suggested by Woolf (1965). Only point (*d*), which indicates that a less extreme configuration has been reached, argues for an earlier stage. However, there is no real substance to that argument, because the mass ratio reached by the end of the rapid phase depends crucially on the initial parameters of the system—especially on the initial mass ratio (Plavec 1968). On the other hand, according to point (*c*) above, the rapid phase must certainly now be over in V356 Sgr, whereas both spectroscopic evidence and period-change evidence show that it is not yet over in β Lyr. Points (*a*) and (*b*) show that the circumstellar disk, which formed around the present primary (original secondary) star as a result of mass accretion, has had some time to be accepted by the primary and has become smaller than it once was, in both radius and thickness. As shown by Figure 3, period-change information is very scanty. However, it seems clear that an enormous dP/P of the order found in β Lyr can be ruled out. Thus it is certain that the rapid phase has ended.

Given that the rapid phase is over, consider the possibility that the system might be in a brief interval of slow mass exchange. The present period (8.9 days) is so much longer than the upper limit (~ 2 days, pre–mass exchange) for case A mass exchange (Plavec 1968) that we must be dealing with case B. In case B we expect only a very short interval ($\sim 10^5$ years [Iben 1967]) of slow mass transfer before helium ignition in the core of the mass-giving component causes that star to detach from its Roche lobe. In some binaries this phase could be absent, if the even briefer rapid phase should begin not very long before helium ignition. During this interval of slow mass transfer only a trivial fraction of the system's mass would be transferred, and for this reason the phase has evoked little interest. However, if it should be present, it would serve as an extremely precise indicator of the evolutionary stage of the binary, and thus would permit virtually exact matching between evolutionary models and the state of the real binary. Probably helium ignition has already occurred, and currently there is no mass transfer at all. However, the fact that the lower-mass star is approximately the size of its Roche lobe and a necessarily short-lived remnant (the disk) of the mass-exchange process can still be seen means that the stage of evolution is quite accurately identified. That is, if helium ignition has occurred, it occurred very recently, because the subgiant component has not contracted very much below its Roche-lobe radius. In view of the foregoing ideas, it would be worth a thorough effort to determine whether any mass exchange at all is now occurring in the system. Period-change information should be interpreted conservatively in this regard, since a lengthening period could result from tidal braking of the more massive star, rather than from mass exchange. A careful spectroscopic search for emission-line activity thus seems the best way of attacking this problem.

Stothers and Lucy (1972) proposed that the primaries of V356 Sgr and μ^1 Sco are examples of stars with substantial underluminosity due to fast differential rotation, induced by mass transfer. Stothers (1973) then calculated quantitatively the angular momentum content of these two stars (plus V Pup and SX Aur) which is implied by their underluminosities. This was possible because, according to Bodenheimer (1971), rotational underluminosity depends only on total rotational angular momentum and not on its distribution within a star. For the V356 primary, Stothers adopted $\delta M_{\mathrm{bol}} = 1.6$ mag, which he found from the effective temperature (i.e., spectral type) and from Popper's (photometric solution) radius of 4.9 $R_{\odot}$. The observational M_{bol} was then compared with Stothers's theoretical mass-luminosity relation for nonrotating zero-age main sequence (ZAMS) stars to find δM_{bol}. From the present solution, which we believe to be an improvement on Popper's, we find a radius of 6.5 $R_{\odot}$, which reduces the underluminosity to 1.0 mag, since the luminosity is given by

$$L/L_{\odot} = (R/R_{\odot})^2 (T_e/T_{e_{\odot}})^4 \,.$$

This residual underluminosity may be real, but we note that it could be removed entirely by a change in spectral classification or a recalibration of the T_e–spectral type relation amounting to two spectral subclasses (i.e., B1 instead of B3). In fact, the B3 type adopted by Popper (1955) included a roughly estimated large correction for the influence of the light of the A-type secondary on the observed type of the primary. The type without this correction was B5–B7.

Plavec (1967) has plotted the V356 Sgr primary in a mass-luminosity diagram as an appropriately large error rectangle. Our results merely move the best estimate of the correct position from the center of the rectangle to the upper part (thus closer to the normal mass-luminosity relation). It is therefore *not* clear that the star is significantly underluminous for its mass. About all one can say is that it would be difficult to find it overluminous. The H-R diagram locations for components 1 and 2 are shown in Figure 5. Component 1 lies to the right of the main sequence, so that, if it has normal luminosity for its mass (cf. discussion above), one would say that it is cool, or has a large radius, for its luminosity. Presumably, it is now settling onto the main sequence after having assimilated most of the mass (all but the present disk) transferred from component 2.

The available information on the rotation rate of the primary requires some comment. Olson (1977) states that $V_1 \sin i$ is 290 km s^{-1} for He I λ4026, 230 km s^{-1} for He I λ4471, and 120 km s^{-1} for Mg II λ4481. We adopt a mean of 260 km s^{-1}, thus disregarding the Mg II result, which should not be characteristic of the B-star surface. The corresponding angular rotation rate would then be about 7 times the synchronous rate (for $R = 6.5\,R_\odot$ and $\sin i \approx 1$), while the theoretical limiting rate from Limber's (1963) Table 1 [for r_1(point) ≈ 0.16] would be about 13 times the synchronous rate. However, note that the disk in our model masks the equatorial, fastest rotating, regions, so that the true $V_1 \sin i$ must be larger than that measured. Thus it seems quite plausible that the surface of the primary is spinning at its maximum possible velocity. Notice that, in order to have the maximum surface rotation rate, the effective gravity need not be zero at a general point on the equator but only at the "balance point" along the line of centers. This balance point is the analog of the inner Lagrangian point in the synchronous Roche problem. Incidentally, Olson's $V_2 \sin i$ of 74 km s^{-1} agrees with that expected for a synchronously rotating secondary.

As a final interesting observational datum, Olson (1977) says: "*Curiously, the B star seems to show a broad stellar K line*" (italics added). Such a feature is entirely foreign to an early B star, but it would be a natural sign of our circumstellar disk.

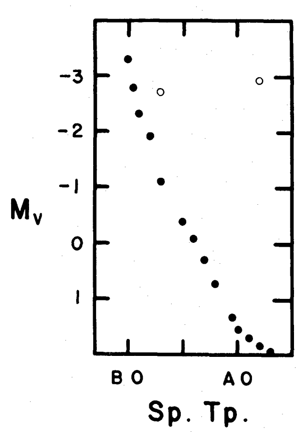

FIG. 5.—An H-R diagram for the components of V356 Sgr (*open circles*). The ZAMS (*filled circles*) is taken from Morton and Adams (1968).

VI. CONCLUSIONS

V356 Sgr cannot be understood as a normal, uncomplicated example of an Algol-type system. We have tried to demonstrate that its peculiarities can most reasonably be explained in terms of a geometrically and optically thick disk which surrounds the (present) primary component. We have modeled this disk quantitatively and found its dimensions by an impersonal adjustment procedure, although computer time restrictions dictated that we forego inclusion in detail of the effects of faster-than-synchronous rotation by the primary star. While the observational effects of the disk are more subtle than those of the β Lyr disk, we believe that its existence is beyond reasonable doubt.

The disk we have found is in good accord with present ideas of binary star evolution, and it fills a gap in observed examples of mass transfer. It is the missing link between a system like β Lyr (still in the rapid phase of mass transfer, with an accretion disk that virtually fills the target Roche lobe) and a post–mass-transfer system (with no disk, no mass transfer, and a helium-star secondary). Evidence that mass transfer has only recently stopped in V356 Sgr is provided by (*a*) the fact that the secondary is about the size of its Roche lobe, although no mass-transfer effects are evident; (*b*) the fact that the radius of the primary is too large for the ZAMS, which indicates that it may still be settling down to the main sequence, after assimilation of the bulk of the mass flow; (*c*) the possible presence of a small orbital eccentricity, which has not had time to be tidally damped (see remarks by Paczyński 1971, pp. 188–189); (*d*) the presence of the disk, which has not finished accreting onto the primary; and (*e*) the very rapid rotation of the primary, which has not yet been tidally damped

It is important to point out the distinction between the present type of disk and the disklike mass flows in systems like RW Tau and U Cep. Geometrically thick, quasi-permanent disks are known to exist with reasonable certainty only in β Lyr and V356 Sgr, although, according to Plavec (1977), there is spectroscopic evidence for such disks in a number of other binaries (see also the paper on SX Cas by Günther 1959). Little theoretical work has been done on their structure, stability, and temporal evolution. For "disks" of the U Cephei type, several investigators have done extensive simulations (Prendergast and Taam 1974; Lubow and Shu 1975); but these are not really disks in the sense of self-sustaining features of the binary system, for they can be detected (by their emission-line activity) only some of the time—evidently when the mass-transfer rate is relatively high (Batten *et al.* 1975). Let us ask the following simple question: Why are the β Lyr and V356 Sgr disks "permanent," rather substantial structures in these two systems, while such disks do not occur in other mass-exchange binaries? We propose that a necessary and sufficient condition for having a thick, fairly massive disk is that the surface of the accreting star be rotating at its limiting velocity (i.e., that for which the effective gravity goes to zero for at least one point on the surface). The mass-exchange process in β Lyr and V356 Sgr then would have proceeded as follows. The mass which was transferred early in the rapid phase was quickly assimilated by the original secondary (present primary), which therefore was spun-up at the surface and to some extent internally. When the surface rotation reached the limiting rate, no further accretion was possible and the accreting material had no alternative but to accumulate into a thick circumstellar disk, which we now see. As angular momentum is drained back into the orbit of V356 Sgr and β Lyr, their disks can gradually be accepted and eventually will entirely be accepted. If we knew only of β Lyr (target star not spectroscopically observed), this scenario would be more speculative than it now is, for in V356 Sgr the accreting star is observed and is found to be rotating very rapidly—perhaps at its limiting surface velocity. On the other hand, the surface of U Cep is known to be rotating at well below its limiting rate, although the rotation is considerably faster than synchronous (Koch, Olson, and Yoss 1965).

Even the present sketchy observations of V356 Sgr show that if fills an important gap in observational binary star evolution. It should well repay the most intense efforts at gathering further information.

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REFERENCES

Batten, A. H., Fisher, W. A., Baldwin, B. W., and Scarfe, C. D. 1975, *Nature*, **253**, 174.
Bodenheimer, P. 1971, *Ap. J.*, **167**, 153.
Bodenheimer, P., and Ostriker, J. P. 1970, *Ap. J.*, **161**, 1101.
Dworak, T. Z. 1977, *Acta Astr.*, **27**, 151.
Günther, O. 1959, *Astr. Nach.*, **285**, 105.
Hall, D. S. 1971, in *IAU Colloquium No. 15, New Directions and New Frontiers in Variable Star Research* (*Kl. Veröff. d. Remeis-Sternwarte Bamberg*, **9**, No. 100, p. 217).
Huang, S. 1963, *Ap. J.*, **138**, 342.
Iben, I. 1967, *Ann. Rev. Astr. Ap.*, **5**, 571.
Koch, R. H., Olson, E. C., and Yoss, K. M. 1965, *Ap. J.*, **141**, 955.
Kruszewski, A. 1966, *Adv. Astr. Ap.*, **4**, 233.
Limber, D. N. 1963, *Ap. J.*, **138**, 1112.
Lubow, S. H., and Shu, F. H. 1975, *Ap. J.*, **198**, 383.
Morton, D. C., and Adams, T. F. 1968, *Ap. J.*, **151**, 611.
Olson, E. C. 1977, private communication.
Paczyński, B. 1971, *Ann. Rev. Astr. Ap.*, **9**, 183.
Plavec, M. M. 1958, *Mém. Soc. Roy. Sci. Liège*, **20**, 411.
———: 1967, *Bull. Astr. Inst. Czechoslovakia*, **18**, 334.
Plavec, M. M. 1968, *Adv. Astr. Ap.*, **6**, 201.
———. 1977, private communication.
Plavec, M. M., and Polidan, R. S. 1976, in *IAU Symposium No. 73, Structure and Evolution of Close Binary Systems*, ed. P. Eggleton, S. Mitton, and J. Whelan (Dordrecht: Reidel), p. 289.
Popper, D. M. 1955, *Ap. J.*, **121**, 56.
———. 1957, *Ap. J. Suppl.*, **3**, 107.
———. 1977, private communication.
Popper, D. M., and Plavec, M. M. 1976, *Ap. J.*, **205**, 462.
Prendergast, K. H., and Taam, R. E. 1974, *Ap. J.*, **189**, 125.
Sahade, J., Huang, S., Struve, O., and Zebergs, V. 1959, *Trans. Am. Phil. Soc.*, Vol. **49**, Part 1.
Stothers, R. 1973, *Pub. A.S.P.*, **85**, 363.
Stothers, R., and Lucy, L. B. 1972, *Nature*, **236**, 218.
Wilson, R. E. 1974, *Ap. J.*, **189**, 319.
———. 1975, *A.J.*, **80**, 719.
———. 1976, *A.J.*, **81**, 383.
Wilson, R. E., and Devinney, E. J. 1971, *Ap. J.*, **166**, 605.
Wong, C. Y. 1974, *Ap. J.*, **190**, 675.
Woolf, N. J. 1965, *Ap. J.*, **141**, 155.

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